Foundations 110 – EXAM Review

Chapter 5 – Systems of Linear Inequalities
Graphing Linear Inequalities in Two Variables

Things to note:
      Dashed lines will represent < or > signs
   Solid lines will represent ≤ or ≥ signs
      All points in the shaded region, if plugged into the equation will be TRUE
   All points in the non-shaded region, if plugged into the equation will be FALSE

1 Answer the following for $2x + 4y \geq 16$

    a) Dashed line or solid line? _____
    b) Rewrite the inequality into slope-intercept form.
    c) Graph the boundary line.
    d) Shade above or below the line (test point).
       Try to use (0,0) where possible.
    e) State domain and range.
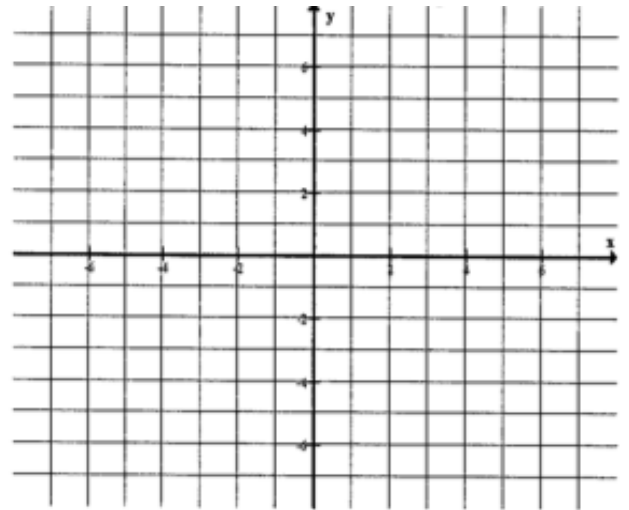
2 Answer the following for $5x + 4y < 20$

    a) Dashed line or solid line? _____
    b) Rewrite the inequality into slope-intercept form.
    c) Graph the boundary line.
    d) Shade above or below the line (test point).
       Try to use (0,0) where possible.
    e) State domain and range.
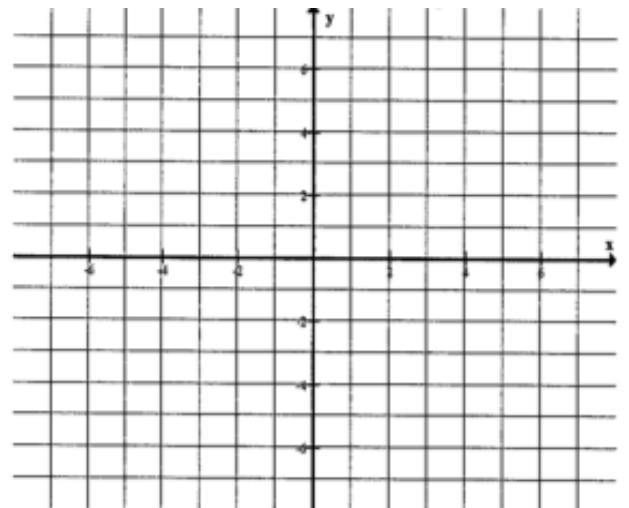
Word Problems

3. Ben is buying snacks for his friends. He has $10.00. The choices are apples for $0.80 and muffins for $1.25.
   a) Write an inequality to model this situation. Define your variables.
   b) State the restrictions on the variables.
   c) Graph the inequality.
   d) Why is (5,4.8) not a possible solution?
   e) State domain and range.
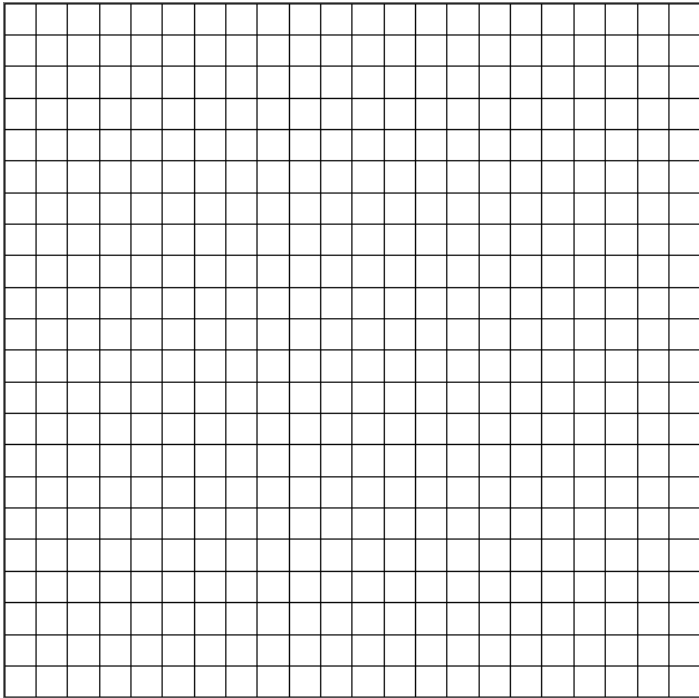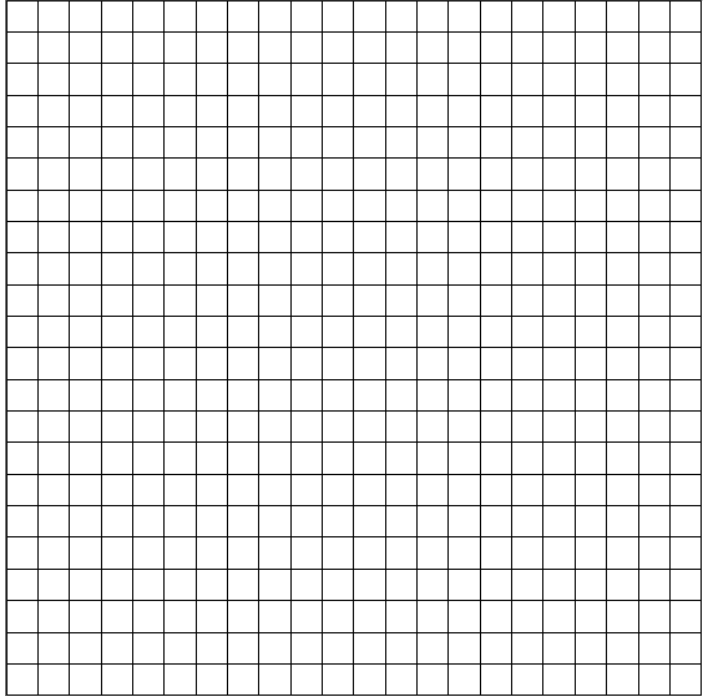
4. A sports store has a net revenue of $100 on every pair of downhill skis sold and $120 on every snowboard sold. The manager's goal is to have a net revenue of more than $600 a day from the sales of these two items. What combinations of ski and snowboard sales will meet or exceed this daily sales goal?

   a) Write an inequality to model this situation. Define your variables.
   b) State the restrictions on the variables.
   c) Graph the inequality.
   d) List two possible combinations that are true and explain what they mean.
   e) State domain and range.

For a system of linear equations, the solution is the point where the two lines intersect.
For a system of linear inequalities, the solution is the region where the shading for each inequality overlaps. The "Feasible Region"

5. Solve the systems of inequalities by graphing.
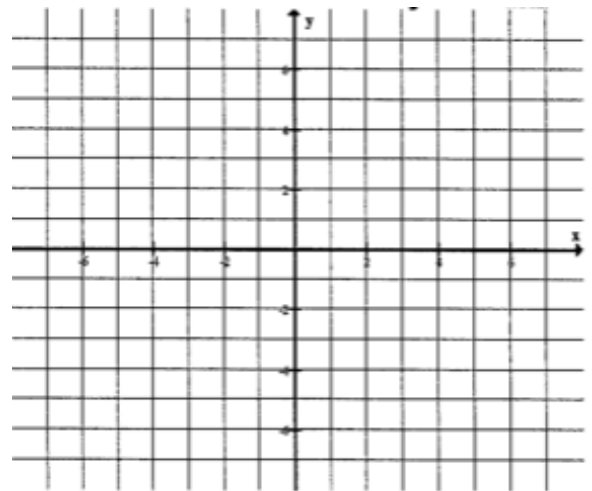
    a) $y \geq 2x - 1$ and $y < x + 5$

What are two possible solutions to this system?
Prove one of them.   (LS I RS Check)
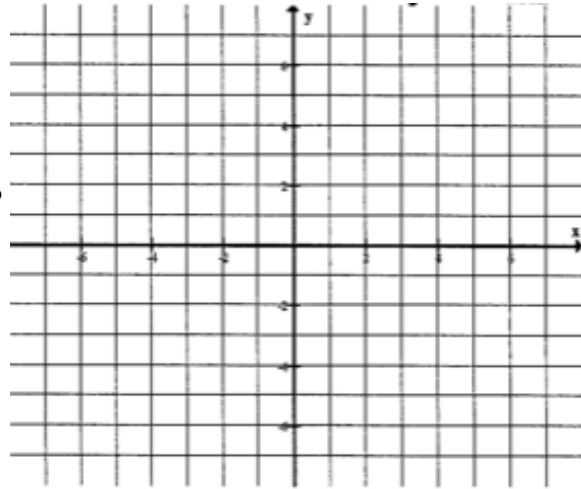
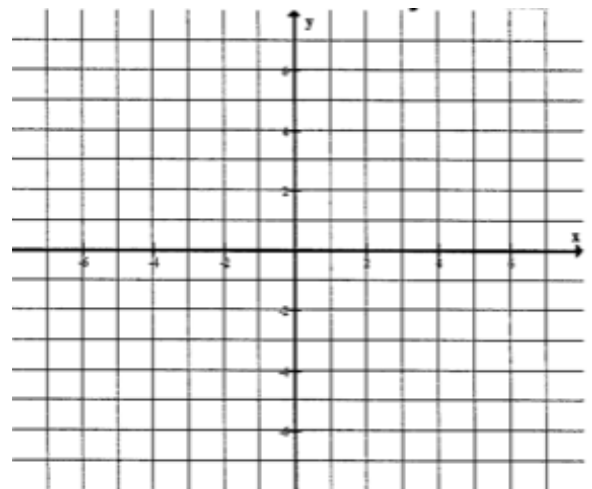   b) $4x - 3y \geq 12$ and $3x + 2y \leq 6$

What are two possible solutions to this system?
 Prove one of them.

6.
$\left\{(x, y) \middle| y \geq -\frac{3}{2}x + 3, x \in R, y \in R\right\}$ and $\{(x, y) | y \geq 6x + 1, x \in R, y \in R\}$

List three points that meet the solution set of this system: _____

Linear inequalities can be used to solve *optimization problems*, problems in which we find the greatest or least value of functions. The method used to solve such problems is called *linear programming*, and consists of two parts:
1. An *objective function* tells us the quantity we want to maximize or minimize.
2. The system of constraints consists of linear inequalities whose region is called the *feasible solution* with area called the *feasible region*.

7. A company makes motorcycles and bicycles. A restricted work area limits the numbers of vehicles that can be made in one day. No more than 10 motorcycles can be made, no more than 15 bicycles can be made, and no more than 20 vehicles of both kinds can be made. If the profit is $25 for a motorcycle and $50 for a bicycle, what should be the daily rate of production of both vehicles to maximize the profits.

Step 1: Identify the quantity that must be optimized.

Step 2: Define the variables that affect the quantity to be optimized and state any restrictions.
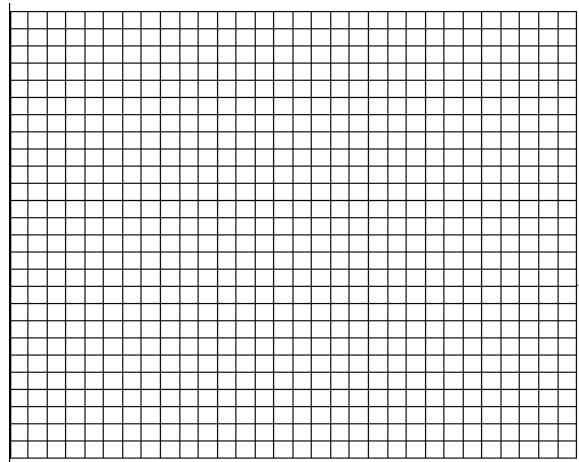
Step 3: Write a system of linear inequalities to describe the constraints of the problem and graph the feasible region.

No more than 10 motorcycles can be made:

No more than 15 bicycles can be made:

No more than 20 vehicles of both kinds can be made:

Step 4: Write an objective function.

8. Fred is planning an exercise program where he wants to run and swim every week. He doesn't want to spend more than 12 hours a week exercising and he wants to burn at least 1600 calories a week. Running burns 200 calories an hour and swimming burns 400 calories an hour. Running costs $1 an hour while swimming costs $2 an hour. How many hours should he spend at each sport to keep his costs at a minimum?

Step 1: Identify the quantity that must be optimized.

Step 2: Define the variables that affect the quantity to be optimized and state any restrictions.

Step 3: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.

Step 4: Write an objective function.
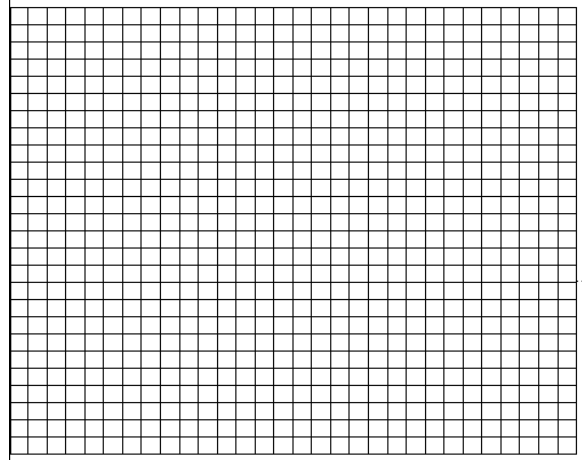
Graphing Quadratic Functions
1. Which functions are quadratic functions? How do you know?
   a) $y = -2x^2 + 3x - 7$
   b) $y = 2x + 3y - 10 = 0$
   c) $y = 2(x - 5)^2 + 10$
   d) $y = 4x^3 - 2x + 5$
   e) $y = 5(x + 2)(x - 7)$

2. Does the parabola $y = -2x^2 + 6x - 5$ open up or down? How do you know?

3. For the given parabola,
   a) State the equation of the axis of symmetry
   b) State the vertex
   c) Is there a maximum or minimum value? What is the value?
   d) State the x- intercepts as ordered pairs
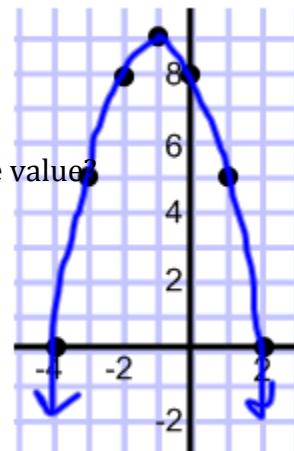
e) State the y-intercept as an ordered pair
f) State the domain of the graph
g) State the range of the graph

4. For each function given:
   - State the direction of opening
   - Use the x and y-intercepts to draw the graph (use a table of values or factor)
   - Find the vertex
   a) $y = x^2 - 3x - 4$
   b) $y = x^2 - 10x + 16$

5. For each function given:
   - Use partial factoring to find two points on the parabola
   - Find the vertex
   - Draw the graph of the quadratic function
   a) $y = -x^2 + 6x - 3$
   b) $y = x^2 + 7x + 4$

6. Given the function $y = (x - 5)(x + 3)$:
   a) State the x-intercepts
   b) State the y-intercept
   c) State the vertex

7. Write the equation of the quadratic function in factored form $y = a(x - r)(x - s)$
   that has x-intercepts of -2 and 5 and goes through the point (10,12).

8. A flare is often used as a signal to attract rescue personnel in an emergency. When a flare is shot into the air, its height, $h(t)$, in meters, over time, t, in seconds can be modeled by
   $h(t) = -5t^2 + 120t$
   a) Identify the x and y intercepts of the parabola
   b) When did the flare reach its maximum height? What was the maximum height?
   c) What was the height of the flare after 15s?
   d) State the domain and range of the function.

9. The equation $y = -4.9x^2 + 20x + 1$ models the path of a skier's jump where y is the height above the ground in meters and x is time the skier was in the air in seconds.
   Determine the maximum height of the skiers jump.

Solving Quadratic Equations

10. Factor each equation and solve for x (Find the roots/x-intercepts)
    a) $x^2 + 7x + 10 = 0$
    b) $x^2 + 3x - 28 = 0$
    c) $x^2 - 4x - 12 = 0$
    d) $2x^2 - 10x = 0$
    e) $3x^2 - 12x + 9 = 0$

11. Use the quadratic formula to solve for x **to one decimal place.** $x =$
    $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
    a) $2x^2 - 5x - 3 = 0$
    b) $x^2 + 4x - 3 = 0$
    c) $3x^2 - 7x + 1 = 0$

12. Solve for x
    a) $y = -2(x + 3)(x - 7)$
    b) $y = (2x + 5)(x - 4) = 0$

13. Samuel is hiking along the top of First Canyon on the South Nahanni River in the Northwest Territories. When he knocks a rock over the edge, it falls into the river, 1260 m below. The height of the rock, h(t), at t seconds, can be modeled by the following function:
    $h(t) = -25t^2 - 5t + 1260$
    a) How long will it take the rock to reach the water?
    b) What is the domain of the function? Explain your answer.

14. A student council is holding a raffle to raise money for a charity fund drive. The profit function for the raffle is $p(c) = -25c^2 + 500c - 350$ where, (c) , is the profit and c is the price of each ticket, both in dollars.
    a) What ticket price will result in the student council breaking even on the raffle?
    b) What ticket price will raise the most money for the school's donation to charity?

Logical Reasoning (Chapter 1)

1. Determine the number that should be in the centre of Figure4.

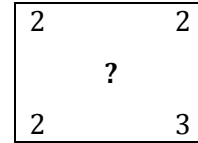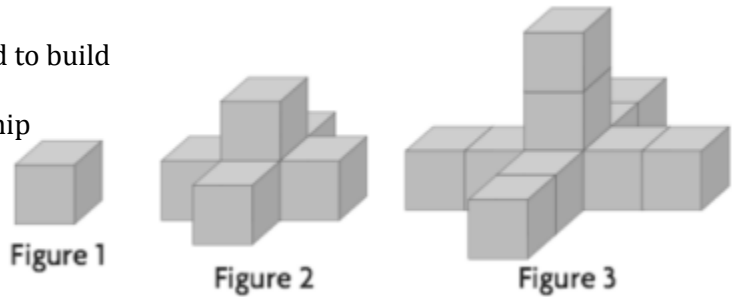| Figure 1	Figure 2	Figure 3	Figure 4
1 ... 2 24 4 ... 3	4 ... 6 192 4 ... 2	3 ... 5 450 6 ... 5	2 ... 2 ? 2 ... 3
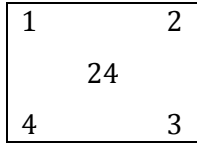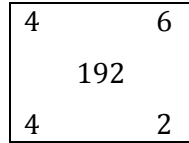
     Figure 1   Figure 2   Figure 3
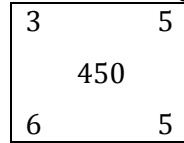
     Figure 4

2. a)Determine the number of cubes needed to build
     Figure 4 and figure 5.
  b) Make a conjecture about the relationship
     between the nth structure and the
     figure number.
  c) How many cubes are needed to
     build the 25th figure?

Figure 1   Figure 2   Figure 3

3. Two mothers and two daughters got off a city bus, reducing the number of passengers by three.
   Explain how this is possible.

4. a) Write a reasonable conjecture about the sum of three odd integers.
   b) Use deductive reasoning to prove that the sum of two even numbers and one odd number will be
     an odd number.

5. Find a counterexample for each of the following conjectures.
  a) When you add a multiple of 6 and a multiple of 9, the sum will be a multiple of 6.
  b) The square of a number is always greater than the number.

6. The three little pigs built three houses: one of straw, one of sticks, and one of bricks. By
   reading the six clues, deduce which pig built each house, the size of each house, and the
   town in which each house was located.
   Clues
       •Penny Pig did not build a brick house.
       •The straw house was not medium in size.
       •Peter Pig's house was made of sticks, and it was neither medium nor small in size.
       •Patricia Pig built her house in Pleasantville.
       •The house in Hillsdale was large.
       •One house was in a town called Riverview.

7. What type of error occurs in the following deduction? Justify your answer.

Let $x = y$.

$x^2 = xy$	Add x^2
$x^2 + x^2 = x^2 + xy$	Simplify
$2x^2 = x^2 + xy$	Subtract 2xy
$2x^2 - 2xy = x^2 + xy - 2xy$	Simplify
$2x^2 - 2xy = x^2 - xy$	Common factors
$2(x^2 - xy) = 1(x^2 - xy)$	Apply $x = y$
$2 = 1$	

**Foundations 110 - Exam Review G1 – Chapter 2**

1. Given that line r is parallel to line s. State the angles that may be referred to as
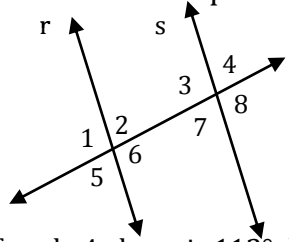
    a) corresponding angles
    b) alternate interior angles
      c) alternate exterior angles
    d) co-interior angles
    e) state the relationship between the angles in a-d

2. a) If angle 4 above is 112° determine the measure of **angle 1 and angle 2**.

   b) In the diagram at right a || b. If ∠4 = 5x + 12 and
     ∠5 = 3x + 24 , find the value of **angle 6**.
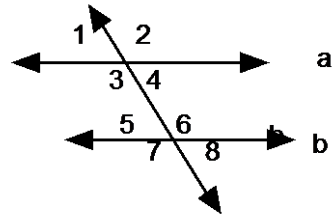
   c) Still referring to the diagram at right. If ∠4 = 3x + 8
     and ∠6 = 5x + 4 , find the value of **angle 2**.

3. Find the value of the angle(s) indicated in each of the following diagrams. Be sure to provide valid
   reasons for each answer.
   a)   b)   c)
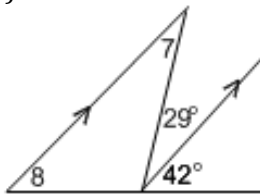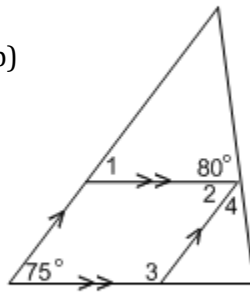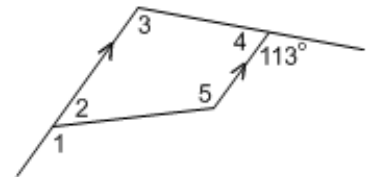
   d) Find the measure of all angles in ΔCDE.   e)
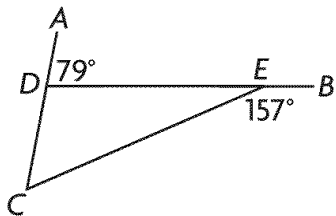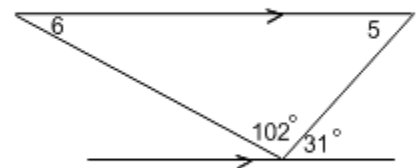
4. Show your work and provide reasons for your answer.
   a) The measure of an interior angle of a regular polygon is 140°. Find the number of sides for the
      polygon.
   b) The measure of an interior angle of a regular polygon is 108°. Find the number of sides for the
      polygon.
   c) The measure of an exterior angle of a regular polygon is 40°. Find the number of sides for the
      polygon.

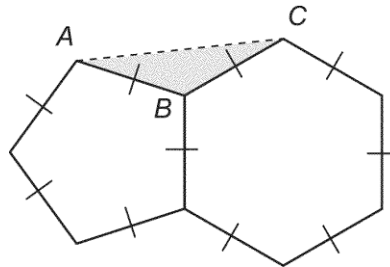 5. Solve for x in each of the following. Provide clear reasons for your answers.

a)

2x + 30°   x

                    130°

b)

        2x

    x

            3x

  2x
                    4x
          7x 3x   8x + 53°
           +17°

c)

106°
            o
      o   2x - 59   o
     o
147ū   x - 3

x + 24   x

d)

115°

                    10x - 17°

                    11x - 26°

118°   6x + 53°
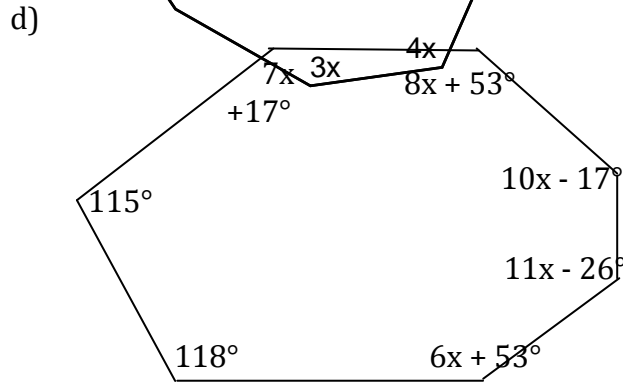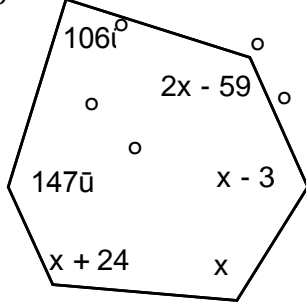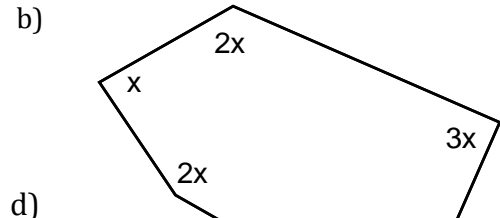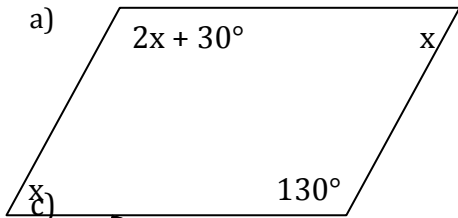
6. Determine the value of d.

a
b
        c   d
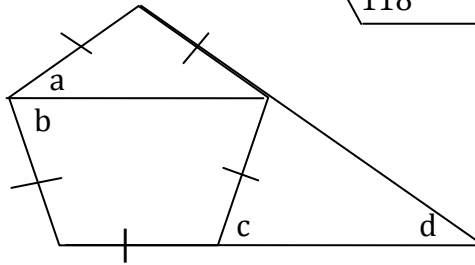
7. A regular hexagon shares a side with a regular pentagon, as shown. Determine the measures of the
   interior angles of ΔABC. Show your solution.

            C
A
    B
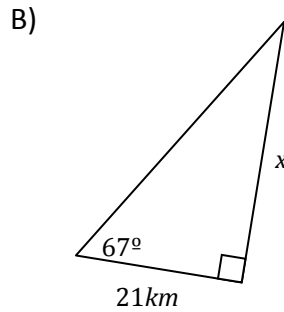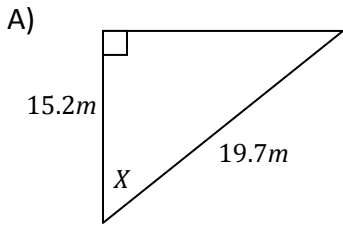
Sine law and Cosine law
Acute and Obtuse Triangle Trigonometry
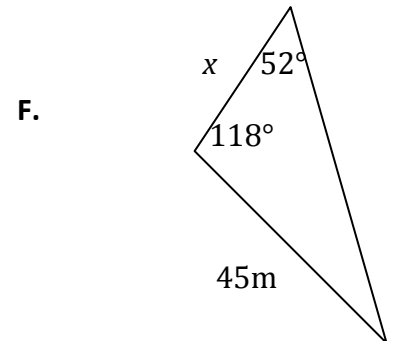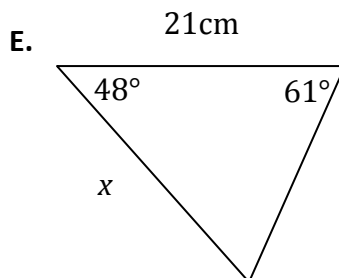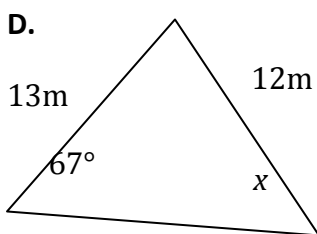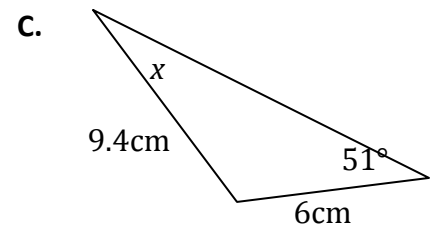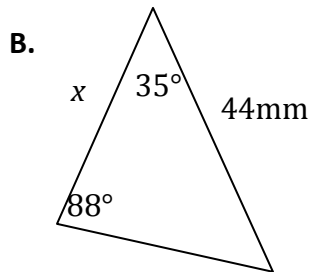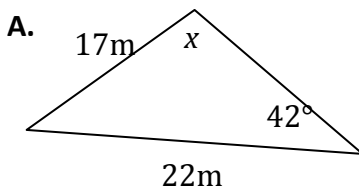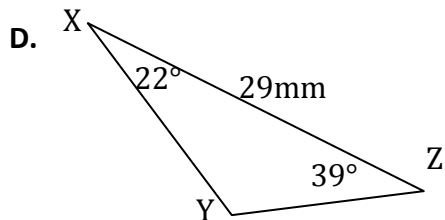1. Solve for the missing side:

A)

16.2*cm*

12.8*cm*

x

B)

22*m*

26*m*

x
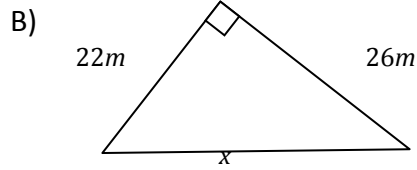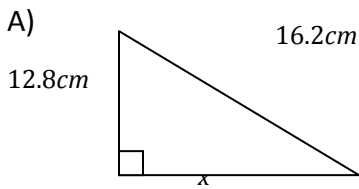
2. Solve for the missing side, *x*, or the missing angle, *X*:
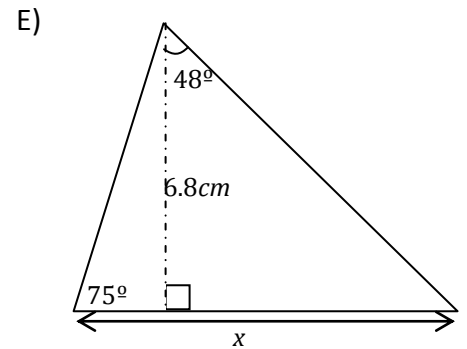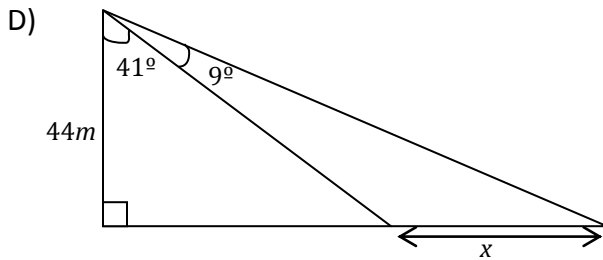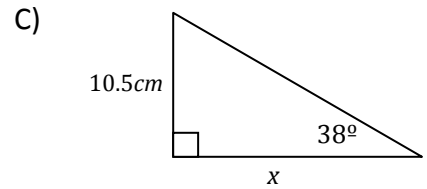
A)

15.2*m*

19.7*m*

X

B)

x

67º

21*km*

C)

10.5*cm*

38º

x

D)

41º 9º

44*m*

x

E)

48º

6.8*cm*

75º

x

3. Solve for the unknown in each triangle. Round to the nearest tenth.

A.

17m *x*

42º

22m

B.

x 35°

44mm

88°

21cm

C.

x

9.4cm

51º

6cm

D.

12m

13m

67°

x

E.

48° 61°
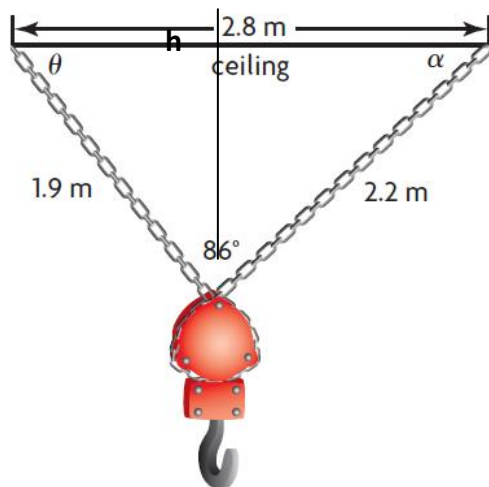
x

F.

x 52º

118°

45m

4.   Solve for **all** missing sides and angles in each triangle. Round to the nearest tenth.

A.

B — 51° ... 71° — A

9.8cm

C

B.

J

42°

K   50m

84°   L

G

C.

R

Q   98°   17.5m

15m   S

M

D.

X

22°   29mm

39°   Z

Y

E.

8cm   115°
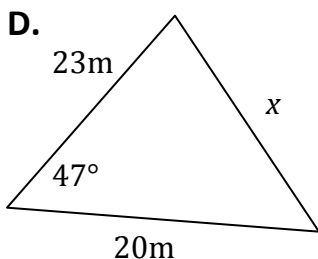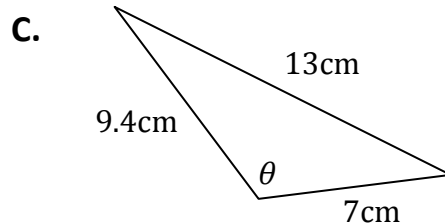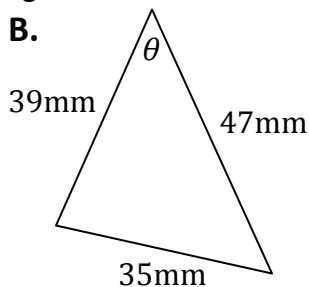
H   I

13cm

F.

28m   31m

N   62°   O
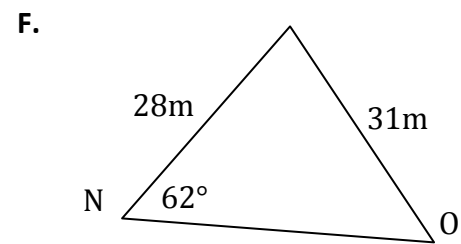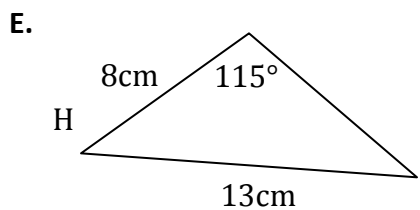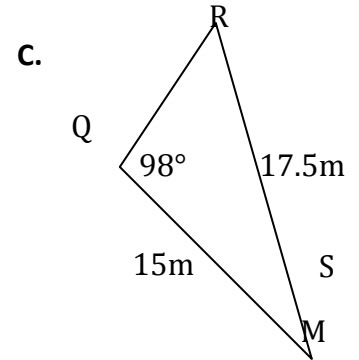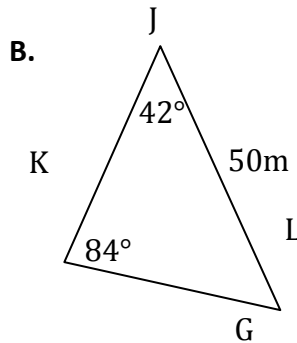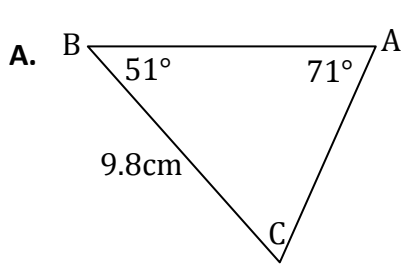
5. Suppose you are the pilot a commercial airliner. You find it necessary to detour around a group of thunderstorms. You turn at an angle of 21° to your original path, fly for a while, turn, and intercept your original path at an angle of 35°, 70 km from your original position. How much further did you go because of the detour?

6. Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch and the ceiling are arranged as shown. How far below the ceiling is the hook?

2.8 m

h

θ   ceiling   α

1.9 m   2.2 m

86°

7. Solve for the unknown in each triangle. Round to the nearest hundredth.

A.

x 17m

42°

22m

B.

θ

39mm   47mm

35mm

C.

13cm

9.4cm

θ

7cm

D.

23m

x

47°

20m

E.
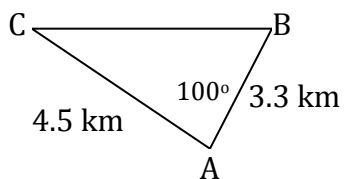
55cm

61°

x 50cm

F.

4.9m   9.1m

8.3m θ
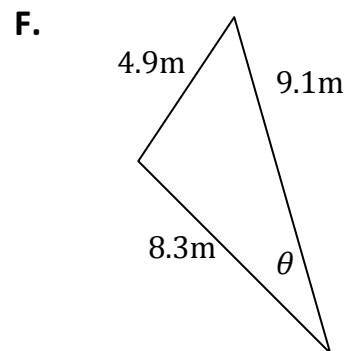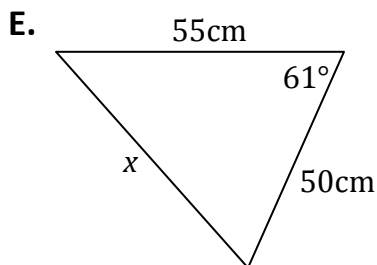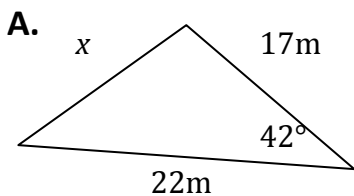
8. Solve for **all** missing sides and angles in each triangle. Round to the nearest hundredth. ** USE PROPER VARIABLES

A. $\Delta XYZ:$ $x = 29m, y = 15m, \angle Z = 122°$

B. $\Delta GHI:$ $g = 13cm, h = 8cm, i = 15cm$

C. $\Delta MNO:$ $n = 31m, o = 28m, \angle M = 62°$

9.) A radar station at A is tracking ships at B and C. How far apart are the two ships?

C   B

100°   3.3 km

4.5 km

A

10. Determine how many triangles are possible in the following situation.

a) A = 55°, a = 7 cm, c = 25 cm

b) A = 55°, a = 27 cm, c = 25 cm

c) A = 55°, a = 22 cm, c = 25 cm

11. Ty and Jake are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Ty's rope is 8.1 m long and makes an angle of 35° with the ground. Jake's rope is 6.1 m long. Assuming they form a triangle, what is the distance between Ty and Jake to the nearest metre.

12. You are setting up a 55 cm long solar panel with a supporting brace that is 22 cm long. The instructions suggest a 15° angle between the ground and the panel. How far from the base of the panel will the supporting brace be?