New outcome : Algebra and Number

AN2 - Operations on radicals and radical expressions with numerical and variable radicands.

index → $\sqrt[n]{a}$ } radical ← radicand

restriction $a \geq 0$

$\sqrt{16}$ $\sqrt[4]{625}$

$\sqrt[3]{8}$

like radicals

$\sqrt{2}$ $5\sqrt{2}$ $-10\sqrt{2}$

$\sqrt[5]{3}$ $7\sqrt[5]{3}$ $-12\sqrt[5]{3}$

similar to polynomials

$5x$ $10x$ $-15x$

x^2 $7x^2$ $-3x^2$

Entire Radicals

Mixed Radicals

Perfect Squares

Converting Entire radicals to Mixed radicals

$\sqrt{24}$ $\sqrt{8}$

.

Convert the following to mixed radicals.

$\sqrt{45}$ $\sqrt{80}$ $\sqrt{72}$

$\sqrt[3]{54}$ $\sqrt[3]{16}$

$\sqrt{108}$ $\sqrt{50}$

.

Convert the following from Mixed to Entire

$4\sqrt{2}$

$2\sqrt{3}$

$6\sqrt{10}$

$2\sqrt[4]{5}$

$3\sqrt[3]{5}$

.

Write as a mixed radical.

1. $\sqrt{12n^3}$

2. $\sqrt[3]{24m^5}$

$\sqrt{-16}$

$\sqrt[3]{-27}$

$\sqrt{27n^9}$

$\sqrt{50n^{15}}$

$\sqrt[3]{125n^7}$

$\sqrt[3]{-40n^4}$

Page 278-9 #2,3,4, 5, 6

Order the following radicals from least to greatest.

$\sqrt{7}$ $\quad 2\sqrt{3}$ $\quad -\sqrt{5}$ $\quad \sqrt{3^2}$ $\quad \frac{\sqrt{16}}{4}$

Adding and Subtracting Radicals

- you must have like radicals to add and subtract radicals

ex. $\sqrt{3}-2\sqrt{5}+4\sqrt{3}+3\sqrt{5}-7\sqrt{10}$

ex2. $\sqrt{2}+7\sqrt{2}+4\sqrt{2}-3\sqrt{7}$

$\sqrt{20}+\sqrt{18}-\sqrt{128}+\sqrt{80}$

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#6,8,9, 10

Multiplying Radicals

-Radicand to radicand
-coefficient to coefficient

$2\sqrt{5}\bullet 3\sqrt{7}$

$\sqrt{5}\times 2\sqrt{3}$

$\sqrt{12}\times 5\sqrt{2}$

$y\sqrt{6y}\left(2\sqrt{7y}\right)$

$\sqrt{2}\left(\sqrt{10}+7\right)$

page 289 #1a-d, 2, 3ab, 4abc 5a

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Dividing Radicals

* Divide coefficents if possible

* Divide radicands if possible

1. $$\frac{12\sqrt{20}}{4\sqrt{5}}$$

2. $$10\sqrt{\left(\frac{36}{25}\right)}$$

3. $$\frac{10\sqrt{12}}{5\sqrt{2}}$$

4. $$\frac{10\sqrt{12}}{5\sqrt{2}}$$

p 290 #6,7

.

We do not want to have radicals in the denominator

We need to rationalize the denominator to eliminate the radical

$$\frac{2}{\sqrt{7}}$$

a) $\frac{5}{\sqrt{5}}$

b) $\frac{5}{\sqrt{3}}$

c) $\frac{2\sqrt{3}}{\sqrt{5}}$

d) $\frac{4\sqrt{2}}{3\sqrt{11}}$

page 290 #8, 15

More Multiplying...

a) $\sqrt{3}\left(\sqrt{3}+2\right)$

b) $\left(5\sqrt{2}-1\right)\left(\sqrt{2}+1\right)$

c) $\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)$

d) $\left(2\sqrt{3}+5\right)\left(2\sqrt{3}-5\right)$

Multiplying by conjugate...

Conjugate:

$\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)$

$\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)$

$\left(2\sqrt{5}+2\right)\left(2\sqrt{5}-2\right)$

.

Rationalize using conjugates...Rationalize the denominator!

$$\frac{4}{\sqrt{2}+1}$$

$$\frac{\sqrt{4}}{2\sqrt{3}-2}$$

$$\frac{\sqrt{5}+1}{2\sqrt{5}-3}$$

You try....

a) $\frac{5}{2\sqrt{5}+3}$

b) $\frac{2\sqrt{2}}{\sqrt{5}+4}$

page 290 #9, 10, 11, 13

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AN3 - Solving Problems that involve radical equations

When solving radical equations, always state restrictions first

Verify all solutions in the original equation to check for extraneous roots

$$5+\sqrt{(2x-1)}=12$$

→ Isolate the radical

State the restrictions...

$\sqrt{(3x)}$

$\sqrt{-5x}$

$\sqrt{6x+9}$

$\sqrt{-4x+6}$

.

Ex 2

$2 \bullet \sqrt{3x-2} = 12$

Solve: $\sqrt{x-3} + 7 = 5$

An extraneous root is a solution found when solving algebraically that is found not to be a solution when checking the result by substitution.

Page 300 #,2,3,4

Solve

$$3\sqrt{2x-4}-4=2$$

Solve

$$\sqrt{2x+7}-x=-4$$

Solve

$m - \sqrt{2m+3} = 6$

page 300-301 #5,6ab,7abd, 8bc, 10ab

Rational Exponents

First, what is a rational number?

A rational number is a number that can be written as a ratio of two integers.

$\frac{a}{b}$ where a and b are integers

Two types of rational numbers

Terminating decimals

Repeating decimals

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.

Rational exponents:

Rational exponents are exponents that are rational numbers, rather self-explanatory, right?

Ex's

There is another way to express a rational exponent, using radicals. The most common radical is the square root.

$\sqrt{x}$ =

In general there are two types of rational exponents:

Simple Roots, where:

When the denominator is 2 we have $b^{1/2} = \sqrt[2]{b} = \sqrt{b}$, the square root, where we normally don't write the 2.
When the denominator is 3, we have $b^{1/3} = \sqrt[3]{b}$ which is referred to as the cube root.

Complex roots where:

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

Convert to radical form:

$c^{\frac{4}{5}}$ $3^{\frac{5}{3}}$

Convert to exponent form:

$\sqrt[3]{m^4}$ $\sqrt[10]{r^9}$

.

PRACTICE

124. Express using rational (fractional) exponents.

a. $\sqrt[3]{5}$ b. $\sqrt{3}$ c. $\left(\sqrt[3]{6}\right)^2$ d. $\sqrt[4]{2^3}$ e. $\sqrt{x^3}$ f. $\sqrt[5]{x^2}$

g. $\sqrt{7^3}$ h. $\left(\sqrt[4]{2}\right)^2$ i. $\sqrt[7]{10}$ j. $\left(\sqrt[8]{x}\right)^4$ k. $\sqrt[4]{3^6}$ l. $\sqrt{x^6}$

125. Express in radical (root) form.

a. $7^{\frac{1}{2}}$ b. $5^{\frac{1}{3}}$ c. $4^{\frac{2}{3}}$ d. $3^{\frac{3}{2}}$ e. $x^{\frac{3}{2}}$ f. $\left(\frac{1}{2}\right)^{-\frac{4}{3}}$

g. $2^{\frac{1}{4}}$ h. $x^{\frac{1}{2}}$ i. $6^{\frac{3}{4}}$ j. $(-3)^{\frac{2}{3}}$ k. $\left(\frac{1}{x}\right)^{-\frac{1}{3}}$ l. $\left(\frac{8}{3}\right)^{-\frac{2}{5}}$

Evaluate each of the following.

a. $25^{\frac{1}{2}}$ b. $8^{\frac{1}{3}}$ c. $9^{\frac{3}{2}}$ d. $64^{\frac{2}{3}}$

126. Evaluate without technology.

a. $49^{\frac{1}{2}}$ b. $27^{\frac{1}{3}}$ c. $\sqrt[3]{2^6}$ d. $\left(\frac{9}{4}\right)^{\frac{3}{2}}$ e. $\sqrt[3]{125^2}$ f. $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$

g. $8^{\frac{2}{3}}$ h. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$ i. $100^{-\frac{3}{2}}$ j. $\left(\sqrt{\frac{16}{25}}\right)^2$ k. $\sqrt[4]{81^3}$ l. $\left(\sqrt[5]{\frac{27}{8}}\right)^{-\frac{5}{3}}$

127. Simplify without technology.

a. $(27^{\frac{1}{2}})^{\frac{2}{3}}$ b. $(16^{\frac{1}{3}})^{\frac{3}{2}}$ c. $\left[\left(\frac{1}{27}\right)^{-2}\right]^{\frac{1}{3}}$ d. $\left[\left(\frac{16}{9}\right)^{-3}\right]^{-\frac{1}{2}}$

e. $\left(\frac{x^4}{9}\right)^{\frac{3}{2}}$ f. $4^{\frac{3}{2}} \times 8^{\frac{1}{3}}$ g. $\left(\sqrt{x^3}\right)\left(\sqrt[4]{x^2}\right)$ h. $2^{\frac{1}{2}} \times 8^{\frac{1}{2}}$

i. $\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$ j. $(9x)^{\frac{3}{2}} \div \left(81x^2\right)^{\frac{1}{4}}$ k. $\left(\sqrt{x}\right)\left(\sqrt[3]{x}\right) \div \left(\sqrt[4]{x}\right)$ l. $\frac{\sqrt[3]{27^2}}{\sqrt[4]{9^6}}$

128. Simplify without technology.

a. $\left(\frac{81}{16}\right)^{\frac{3}{4}} - \left(\frac{9}{4}\right)^{\frac{3}{2}}$ b. $\left(\frac{8}{27}\right)^{\frac{-2}{3}} - \left(\frac{1}{2}\right)^2 + 4^0$ c. $\left(\frac{1}{2}\right)^{-3} + 9^{\frac{1}{2}} - 64^{\frac{2}{3}}$

d. $8^{\frac{2}{3}} + \left(\frac{1}{4}\right)^{-2} + 3^0$ e. $\frac{\left(\frac{1}{2}\right)^{-3} + 27^{\frac{2}{3}} - 5^0}{4^{\frac{1}{2}}}$ f. $\frac{\left(\frac{1}{3}\right)^{-2} + 8^{\frac{2}{3}} + 4^0}{16^{-\frac{1}{2}}}$

☆ 129. Simplify.

a. $\frac{\left(9^{2n-1}\right)\left(27^{n-1}\right)}{\left(3^{n-1}\right)^{-2}}$ b. $\frac{\left(3^{2x}\right)\left(9^{x-1}\right)\left(27^{x+1}\right)}{9^x}$ c. $\frac{\left(25^{3-2x}\right)\left(5^{3x-7}\right)\left(125^{x+1}\right)}{25^{x+2}}$

d. $\frac{\left(8^n\right)\left(2^{n-2}\right)\left(4^{n+1}\right)\left(3^n\right)}{\sqrt{3^{2n}}}$ e. $\frac{\left(2^x\right)\left(8^{x+2}\right)\left(32^{2x-1}\right)}{\left(8^x\right)\left(\sqrt{4^x}\right)}$ f. $\frac{\left(\sqrt[3]{8^{3x}}\right)\left(\sqrt{4}\right)^{x-3}\left(16^{3x+1}\right)}{\left(\sqrt[5]{32^2}\right)^{x+1}\left(4^{x+2}\right)}$