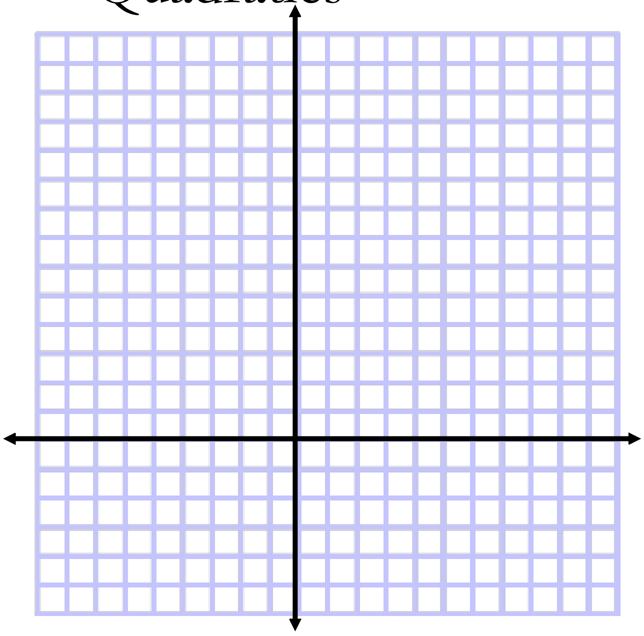
Quadratics

$$y = x^2$$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Groups of 4:

For your equations:
a) make a table of values
b) plot the graph
c) identify and label the:
- i) vertex
- ii) Axis of symmetry
- iii) x- and y-intercepts

Group 1:

$$y = (x-3)^2$$
$$y = (x+5)^2$$
$$y = (x-1)^2$$

Group 2

$$y = x^2 - 3$$
$$y = x^2 + 2$$
$$y = x^2 + 1$$

Group 3

$$y = 2x^2$$
$$y = \frac{1}{2}x^2$$
$$y = -3x^2$$

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What is the effect of the following:

$$y = ax^2$$

$$y = x^2 + k$$

$$y = (x - h)^2$$

$$y = -x^2$$

Transformations of Quadratics Functions

$$y = a(x - p)^2 + q$$ vertex form

a -

q -

p -

"-

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Transformations of Quadratic Functions

RF3 - Analyze quadratic functions of the form $y = a(x-p)^2 + q$
Determine the vertex, domain and range, direction of opening, axis of symmetry, x and y intercepts

1. Determine a rule for each transformation

A. $y = ax^2$

$y = -x^2$ $\quad y = \frac{1}{2}x^2$

$y = 2x^2$ $\quad y = -\frac{1}{2}x^2$

$y = -2x^2$

B. $y = x^2 + q$

$y = x^2 + 4$ $\quad y = x^2 + 1$

$y = x^2 - 3$ $\quad y = x^2 - 5$

C. $y = (x-p)^2$

$y = (x-3)^2$ $\quad y = (x+4)^2$

$y = (x+1)^2$ $\quad y = (x-2)^2$

3. Put it all together: $y = a(x-p)^2 + q$

Use your conclusions from #1 to state the vertex and the direction of opening for each function

$$y = 2(x-3)^2 + 4$$
$$y = -3(x+1)^2 - 5$$

2. For each function: state the vertex, axis of symmetry and the maximum/minimum value

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4. How many x-intercepts will each function have?

$y=(x-5)^2-7$ $\quad$ $y=2(x+7)^2+3$ $\quad$ $y=-3(x+2)^2$

5.

Function	vertex	range	axis of symmetry	direction of opening	x int's?
$y=(x-2)^2+3$					
$y=-x^2-3$					
$y=(x+5)^2$					
$y=-4(x+1)^2-3$					

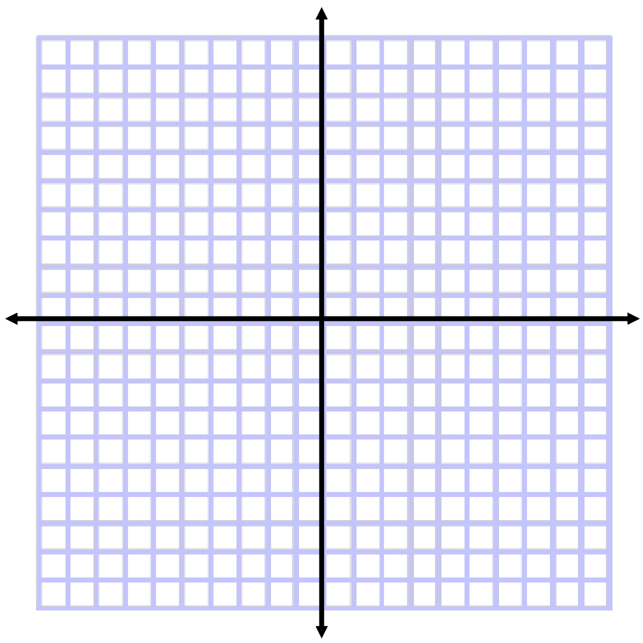
6. Use transformations to sketch each function

$y=(x+3)^2-2$

$y=2(x-1)^2+3$

$y=-3(x+2)^2+1$

$y=\frac{1}{2}(x+4)^2$



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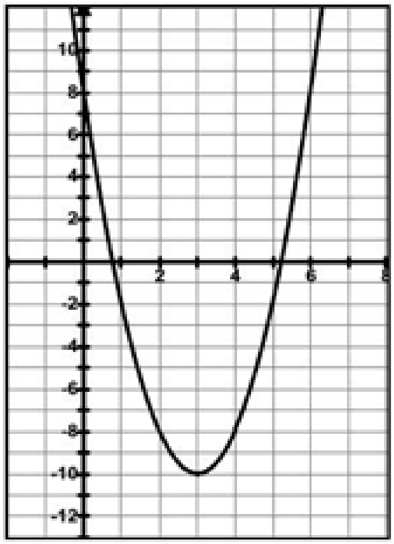
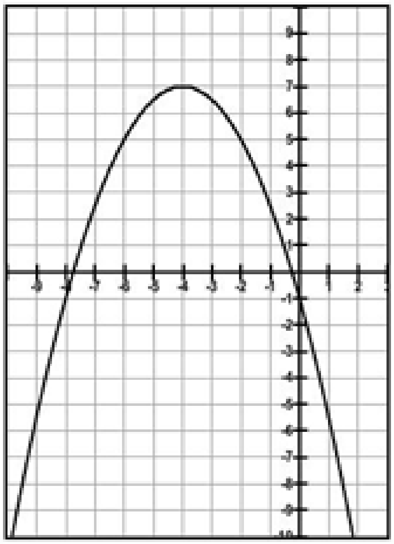
Determining the equation of a quadratic equation.

EXAMPLE: Examine the following graphs and identify the equation of each function.

A.



B.



a. Identify the vertex. ⇨ ________

b. Identify the axis of symmetry. ⇨ ________

c. Identify the y-intercept. ⇨ ________

d. Identify the vertical stretch. ⇨ ________

a. Identify the vertex. ⇨ ________

b. Identify the axis of symmetry. ⇨ ________

c. Identify the y-intercept. ⇨ ________

d. Identify the vertical stretch. ⇨ ________

As you can see, using the characteristics of a quadratic function:

Vertex (p, q)
Axis of Symmetry x=p
Vertical Stretch a

We can write the equation in vertex form

$$y = a(x-p)^2 + q$$

The most challenging characteristic to find is the vertical stretch.This value can be determined if we know the vertex and one other point.

A golf ball is hit from the fairway with a high chip shot. It reaches a maximum height of 20 m and lands on the green 10 m away. Determine the equation that describes the golf ball's path.

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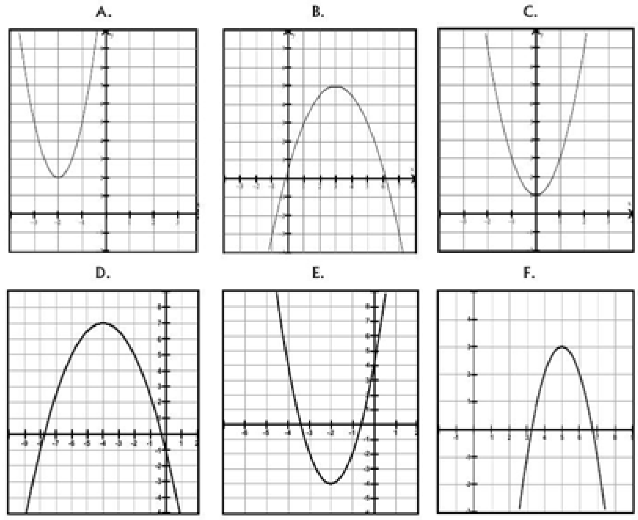
Write the equation of the following in vertex form:

1.



2. Find the vertical stretch and write the equation in vertex form.

a. vertex (2, 5) and has a y-intercept of 3

b. vertex (6, −2) and has a y-intercept of −8

c. vertex (4, 3) and has x-intercepts 2 and 6

d. vertex (−2, −4) and has x-intercepts −4 and 0

3. A rock is thrown into the air from an initial height of 2 metres. After 2 seconds it reaches a maximum height of 10 metres. Determine the equation of the quadratic function that describes the path of the rock.

3. A wedding arch is in the shape of a parabola. If the arch is 2 m wide and 3 m tall, determine the equation that describes the shape of the arch.

4. An arrow is fired into the air and reaches a maximum height of 30 m at a horizontal distance of 50 m from where it is fired. It sticks in the ground 90 m away from where it is fired.

a) determine the equation of the quadratic function that describes the path of the arrow.

b) How high is the arrow after travelling a horizontal distance of 80 m?

5. A football is kicked for a field goal attempt and it reaches a maximum height of 25 m at a horizontal distance of 20 m.

a) Determine the equation of the quadratic function that describes the path of the football.

b) If the field goal marker is 35 m away at a height of 3 m, would the kick score the points?

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RF4. Analyze quadratic functions of the form to identify characteristics of the corresponding graph, including: vertex, domain and range, direction of opening, axis of symmetry, x**- and** y**-intercepts; and to solve problems**. [CN, PS, R, T, V]

Vertex form: $y = (x-2)^2 + 7$

Expanded:

Standard Form:

Going backwards, we need to use a process called completing the square to return (or to convert) to vertex form

$$y = x^2 - 4x + 11$$

we need to make y=x² - 4x+____ part of a perfect square trinomial

$$y = x^2 + 6x + 5$$

$$y = x^2 - 10x + 12$$

.

$y = x^2 - 5x + 1$

More completing the square!

$y = -x^2 + 6x + 7$

When a≠1, you need to group the first two terms and factor the leading coefficient out.

$y = 3x^2 + 12x - 5$

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2. Complete the square and find the vertex!

$$y = -x^2 - 6x + 2$$

$$y = x^2 + 5x - 2$$

page 192-3 #2ab, 3ab, 4ab, 5ab, 6ab, 7ab, 9, 12ac

Using completing the square to derive the quadratic formula:

$$y = ax^2 + bx + c$$

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Complete the square to write in vertex form

1. $y = x^2 + 16x - 2$

2. $y = x^2 - 7x - 5$

3. $y = -x^2 - 14x + 3$

4. $y = 4x^2 - 12x + 7$

5. $y = -3x^2 + 18x + 1$

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SCO: RF5. Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]

Solving a Quadratic Equation

Roots of an equation

zeros of a function

x-intercepts of a graph

These all mean to let y=0 and solve for x

Methods used to solve quadratic equations

1. Square root

$$x^2 - 49 = 0$$

2. Factor

$$x^2 + 3x - 10 = 0$$

3. Complete the square

$$x^2 - 6x + 1 = 0$$

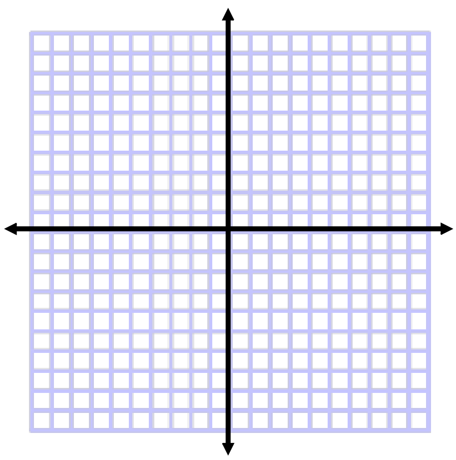
4. Quadratic formula

$$x^2 - 6x + 1 = 0$$

5. Graph

$$(x-2)^2 - 4 = 0$$

$$x^2 - 4x = 0$$



Show that the following quadratic equations are equivalent

standard form

$$y = x^2 + 10x + 7$$

vertex form

$$y = (x+5)^2 - 18$$

A Quadratic Equation has two roots

standard form

$$y = ax^2 + bx + c$$

vertex form

$$y = a(x-p)^2 + q$$

factored form

$$y = a(x-r)(x-s)$$

What kind of roots will a quadratic function have?

$$y = (x+2)^2 - 5$$

$$y = (x-3)^2$$

$$y = (x-1)^2 + 3$$

,

How do you determine the number of roots when in standard form?

$y = x^2 + 4x - 1$ $y = x^2 - 6x + 9$ $y = x^2 - 2x + 5$

The Discriminant tells you what type of roots the quadratic function will have

if D>0 if D=0 if D<0

$y = x^2 + 4x + 2$ $y = x^2 - 10x + 25$ $y = x^2 + 2x + 5$

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